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# TEMPERATURE FIELD OF LAMINAR-INHOMOGENEOUS BEDS

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Results are presented of a theoretical investigation of the temperature field of oil beds with a nonuniform structure, with applications in the technology of selective thermoinjection.

Heat transfer accompanying the filtration of liquid in permeable media is the physical basis of many processes in mining, the power industry, and chemical engineering.

The formulation of multidimensional problems is a particularly urgent matter. Such problems include those concerning filtration in an infinite porous medium with point sources and sinks and in a plane porous bed with heat transfer to the roof and floor; problems for collectors of different form, taking free convection into account; etc.

A possible approach to the solution of such problems consists in the use of the idea of "homogenization" of the heterogeneous medium [1], analogously to the methods of the mechanics of interpenetrating media [2] or in the approximation of instantaneous temperature equalization of the two phases [3]. On this basis, fairly many problems may be solved, but in practice they give rise to a series of serious objections. For example, one unsolved problem is the choice of the heat-transfer coefficient at the interface of the two phases, since this is not simply the heat-transfer coefficient between the individual elements of the filling and the liquid, but a coefficient or function which must take into account all the arbitrariness of the given approach. In addition, the transfer coefficients in the equations are not actually the weighted means of the molecular coefficients, as concluded, for example, in [4, 5].

Processes of nonsteady heat transfer accompanying one-dimensional filtration in fillers consisting of small particles with low thermal resistance are usually described using the formulation of the problem first proposed in [6, 7]. However, the results obtained are difficult to use in developing engineering methods of calculation for the heat transfer in more complex multidimensional filtration regions.

In [8], the problem of describing the heat conduction for an analogous physical situation was considered in more detail on the basis of a generalized equation for one dependent variable obtained in [8].

In mining thermophysics (in developing methods of creating systems for the extraction of petrogeothermal resources, and also thermal methods of treating petroleum beds, etc.), it is necessary to develop a method of calculating the heat transfer accompanying filtration in collectors of complex geometry with large structural elements. In constructing the model in this case, it is more correct to use a formulation of the problem in which the finite heat conduction of the elements of the permeable bed is taken into account (see, e.g., [9-11]). A more detailed review of methods of calculating the nonsteady heat transfer accompanying one-dimensional filtration is given in [12].

The use of accurate solutions of heat-conduction problems for particles of the bed and the surrounding rock mass reduces the system of energy equations to an integrodifferential equation, which is not readily generalized to the case of multidimensional filtration. Therefore, in [12], a new approach to the solution of problems of this type was proposed.

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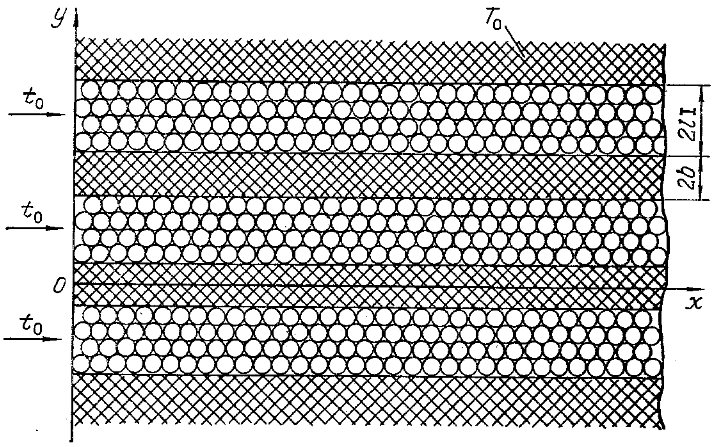


Fig. 1. Physical model of heat transfer in a laminar-inhomogeneous bed.

In the present work, its specific application in describing the temperature field of a petroleum bed of nonuniform structure is demonstrated.

In [13], the technology of selective thermoinjection of laminar-inhomogeneous beds was described, and the results of laboratory investigations were given. There are no analytical solutions for this scheme. A physical model of the problem corresponding to the given technology is shown in Fig. 1.

Hot heat carrier at a temperature $t_0$ is injected into the highly permeable layer, displacing petroleum from it, and simultaneously increasing the temperature of the adjacent, relatively impermeable layers. Some of the heat thus introduced goes to heating of the skeleton of the highly permeable porous layers. The mean over the permeable-layer cross section of the dimensionless temperature of unit volume of liquid will be given by the heat-balance equation

$$c_L \rho_L \frac{\partial \theta}{\partial \tau} + c_L \rho_L u \frac{\partial \theta}{\partial x} = \sigma q_1 + q_2/b, \tag{1}$$

which must be solved together with initial and boundary conditions of the form

$$\theta = 1 \text{ when } x = 0; \quad \theta = 0 \text{ when } \tau^* = \tau - x/u \leqslant 0. \tag{2}$$

Here heat transfer between the phases is taken into account by introducing internal heat sources in the heat-balance equation of the liquid phase, represented by the specific heat fluxes $q_1$ (to the surface of particles in the porous layer) and $q_2$ (to the "impermeable" layers).

The heat fluxes $q_1$ and $q_2$ are determined from the solution of the corresponding heat-conduction problems for a sphere and a plate, taking account of the finite rate of interphase heat transfer and the time-variable temperature of the surrounding medium [14]. Substituting these solutions into Eq. (1) allows the problem in Eqs. (1) and (2) to be rewritten in the form

$$\frac{\partial \theta}{\partial \tau} + u \frac{\partial \theta}{\partial x} = -\left\{ \frac{2\lambda_I c_I}{l_I b c_L \rho_L} \sum_{n=1}^{\infty} \frac{\partial}{\partial \tau^*} \int_0^{\tau^*} \theta(\omega) \exp[-\mu_n^2 a_I (\tau^* - \omega)/l_I^2]\, d\omega + \frac{D_n 2\sigma \lambda_P}{R_P c_L \rho_L} \sum_{n=1}^{\infty} \frac{\partial}{\partial \tau^*} \int_0^{\tau^*} \theta(\omega) \exp[-\beta_n^2 a_P (\tau^* - \omega)/R_P^2] d\omega \right\}, \tag{3}$$

$$\theta = 1 \text{ when } x = 0, \qquad \theta = 0 \text{ when } \tau^* = \tau - \frac{x}{u} \leqslant 0. \tag{4}$$

Using the method of the "equivalent" heat-conduction equation [12], the integrodifferential Eq. (3) (in dimensionless form) is brought to the form

$$\frac{\partial \theta}{\partial \mathrm{Fo}^*} + \frac{1}{A G_I} \frac{\partial \theta}{\partial X} = \frac{B}{A} \frac{\partial^2 \theta}{\partial \mathrm{Fo}^{*2}}. \tag{5}$$

The initial and boundary conditions are

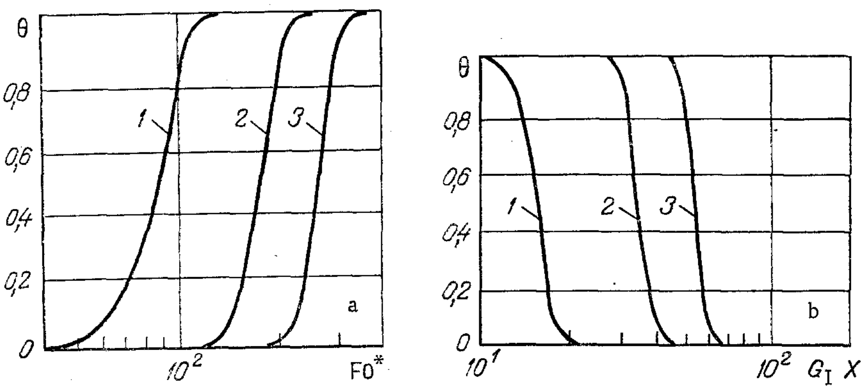


Fig. 2. Temperature change at the center of a relatively impermeable layer over time (a) with $G_I X = 50$ (1), 100 (2) and 150 (3), and over the length (b) with Fo = 70 (1), 150 (2), and 230 (3); Y = 0.

$$\theta = 1 \text{ when } X = 0, \quad \theta = 0 \text{ when } \mathrm{Fo}^* \leqslant 0, \quad \theta = 1 \text{ as } \mathrm{Fo}^* \to \infty. \tag{6}$$

The solution for the liquid temperature is obtained by the method of Laplace transformation, and takes the form

$$\theta = 1 - \frac{1}{2}\left\{\operatorname{erfc}\left[\frac{\mathrm{Fo}^* - AG_I X}{2\sqrt{BG_I X}}\right] + \exp\left(\frac{A}{B}\mathrm{Fo}^*\right)\operatorname{erfc}\left[\frac{\mathrm{Fo}^* + AG_I X}{2\sqrt{BG_I X}}\right]\right\}, \tag{7}$$

where

$$A = A_P \frac{G_P}{G_I} + A_I; \quad B = B_P \frac{G_P}{G_I}\frac{\tau_P}{\tau_I} + B_I.$$

The coefficients $A_P$, $B_P$ and $A_I$, $B_I$ for boundary conditions of the third kind are calculated from the following formulas

$$A_P = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{2\mathrm{Bi}_P^2 \sqrt{\beta_n^2 + (\mathrm{Bi}_P - 1)^2} \sin\beta_n}{(\beta_n^2 + \mathrm{Bi}_P^2 - \mathrm{Bi}_P)\beta_n^2},$$

$$B_P = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{2\mathrm{Bi}_P^2 \sqrt{\beta_n^2 + (\mathrm{Bi}_P - 1)^2} \sin\beta_n}{(\beta_n^2 + \mathrm{Bi}_P^2 - \mathrm{Bi}_P)\beta_n^4},$$

$$A_I = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{2\mathrm{Bi}_I \sqrt{\mathrm{Bi}_I^2 + \mu_n^2} \sin\mu_n}{(\mathrm{Bi}_I + \mathrm{Bi}_I^2 + \mu_n^2)\mu_n^2},$$

$$B_I = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{2\mathrm{Bi}_I \sqrt{\mathrm{Bi}_I^2 + \mu_n^2} \sin\mu_n}{(\mathrm{Bi}_I^2 + \mathrm{Bi}_I + \mu_n^2)\mu_n^4}; \tag{8}$$

$\mu_n$ and $\beta_n$ are found from the solution of the equations

$$\operatorname{ctg}\mu_n = \frac{1}{\mathrm{Bi}_I}\mu_n, \quad \operatorname{tg}\beta_n = \frac{1}{\mathrm{Bi}_P - 1}\beta_n. \tag{9}$$

On the basis of the Duhamel theorem, the solution for the dimensionless temperature of the relatively impermeable layer may be written in the form

$$\vartheta(\mathrm{Fo}^*, X, Y) = \frac{\partial}{\partial \mathrm{Fo}^*}\int_0^{\mathrm{Fo}^*} \theta(\Omega, X)\left\{1 - \sum_{n=1}^{\infty} K_n \cos(\mu_n Y)\exp[-\mu_n^2(\mathrm{Fo}^* - \Omega)]\right\} d\Omega. \tag{10}$$

Transforming the solution in Eq. (10) by means of the results of [12] gives the relation

$$\vartheta(\mathrm{Fo}^*, X, Y) = \theta(\mathrm{Fo}^*, X) - K_1 \frac{\partial\theta}{\partial \mathrm{Fo}^*} + K_2 \frac{\partial^2\theta}{\partial \mathrm{Fo}^{*2}}. \tag{11}$$

The solution for the liquid temperature $\theta$ is known — Eq. (7); the coefficients $K_1$ and $K_2$ are defined by the expressions

$$K_1 = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{2\,\mathrm{Bi_I} \sqrt{\mathrm{Bi_I^2} + \mu_n^2} \cos(\mu_n Y)}{\mu_n^3 (\mathrm{Bi_I^2} + \mathrm{Bi_I} + \mu_n^2)},$$

$$K_2 = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{2\,\mathrm{Bi_I} \sqrt{\mathrm{Bi_I^2} + \mu_n^2} \cos(\mu_n Y)}{\mu_n^5 (\mathrm{Bi_I^2} + \mathrm{Bi_I} + \mu_n^2)}. \quad (12)$$

The temperature distribution in the impermeable layers is shown in Fig. 2 for the following values of the parameters determining the process: $\tau_P = 18 \cdot 10^{-6}$; $\tau_I = 9$; $G_P = 8$; $G_I = 0.4$; $\mathrm{Bi_I} = 0.3$; $\mathrm{Bi_P} = 0.005$. Thus, for example, the time for which hot water of temperature $t_0$ must be pumped through the porous layer in order to heat the "impermeable" layer (Y = 0) to a temperature ensuring a sharp increase in viscosity of the petroleum or a specified increase in the petroleum extraction may be determined from Fig. 2a.

From Fig. 2b, the extent of the "impermeable" layer heated to the technologically required temperature at a specific moment of time is determined. Of course, the curves in Fig. 2 only show an example of a calculation by the proposed formula, and do not cover the whole range of parameters encountered in practice.

## NOTATION

x, y, longitudinal and transverse coordinates; $\tau$, time; t, liquid temperature; T, temperature of layers; $T_0$, initial temperature of the layer; $t_0$, liquid temperature at the inlet; u, actual velocity; b, half-height of highly permeable layer; $l_I$, half-height of relatively impermeable layer; $R_P$, particle radius of layer; $\sigma$, surface area of layer particles per unit volume of injected liquid; $\lambda_I$, $a_I$, effective thermal conductivity and thermal diffusivity of relatively impermeable layers; $\lambda_P$, $a_P$, thermal conductivity and thermal diffusivity of layer particles; $\alpha_I$, heat-transfer coefficient to the walls of the relatively impermeable layers; $\alpha_P$, heat-transfer coefficient to layer particles; $\rho_L$, $c_L$, density and specific heat of liquid; $\rho_I$, $c_I$, density and specific heat of relatively impermeable layers; $\theta = t - T_0/t_0 - T_0$, dimensionless temperature of liquid; $\vartheta = (T - T_0)/(t_0 - T_0)$, dimensionless temperature of relatively impermeable layers. Dimensionless complexes: $\mathrm{Fo}^* = \mathrm{Fo} - X$; $\mathrm{Fo} = a_I\tau/l_I^2$; $X = a_I x/l_I^2 u$; $Y = y/l_I$; $\tau_P = R_P^2/a_P$; $\tau_I = l_I^2/a_I$; $G_I = l_I\rho_I c_I/b\rho_L c_L$; $G_P = R_P\sigma\rho_P c_P/\rho_L c_L$; $\mathrm{Bi_I} = \alpha_I l_I/\lambda_I$; $\mathrm{Bi_P} = \alpha_P R_P/\lambda_P$.

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# COUPLED PROBLEM OF HEAT TRANSFER, HYDRODYNAMICS, AND SOLIDIFICATION IN A MELT

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A mathematical model is constructed which describes thermal and hydrodynamic phenomena accompanying the solidification process in a melt. The equations of hydrodynamics take into account viscoelasticity and compressibility of liquid metal. An example of calculations pertaining to solidification of an ingot is given.

Motion of the melt in the still liquid part of a crystallizing ingot greatly affects the quality of the metal product. Many studies have, therefore, been made concerning convective flow in the liquid core of ingots [1-7].

In several studies [1-6] the equations of heat conduction and of melt motion were solved independently of the solidification problem, i.e., for a given configuration and with the interphase boundaries moving according to a given law. In one study [7] a mathematical model has been proposed which, through a coupled formulation of the problems of hydrodynamics, heat transfer, and solidification, accounts for the interdependence between the form of the crystallization front and the mode of thermogravitational convection developing in the liquid phase of the ingot, both varying in time. A numerical simulation of this model [7] is filled with additional difficulties in connection with satisfying the Stefan condition at a movable and generally curvilinear crystallization front. In this study we will supplement the coupled formulation of those problems with the concept of a two-phase zone [8] and will account for the release of the latent heat of crystallization within this zone by stipulating an effective (apparent) specific heat.

In the previous studies [1-7] the flow of liquid steel was calculated through solution of the system of Navier—Stokes equations. At large temperature drops typical of metallurgical processes, the flow of the melt ceases to be laminar, however, and becomes a nonsteady fluctuating one. Under these conditions, moreover, the liquid can exhibit properties not included in the Navier—Stokes law. We propose to replace the Navier—Stokes equations with equations of motion based on the Maxwell law of viscoelasticity and an equation of state of the medium involving a pressure dependence of the density, i.e., accounting for the compressibility of the medium.

In this way the coupled problem of heat transfer, hydrodynamics, and melt solidification is formulated in three segments:

1) problem of heat transfer involving the liquid phase and the solid phase of an ingot and taking into account the release of heat of phase transition within the liquidus—solidus temperature range $T_l - T_s$;

2) problem of hydrodynamics involving the motion of a compressible viscoelastic liquid in the still unsolidified part of the ingot and taking into account a nonuniform temperature profile as well as the attendant Archimedes body forces;

3) conditions of coupling between the thermal problem and the hydrodynamic problem at the interphase boundary.

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